Effect of Void Volume and Prandtl Modulus on Heat Transfer in Tube Banks and Packed Beds

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A correlation is presented relating the heat transfer characteristics of cross-flow heat exchangers to the void-volume-and-tube-pitch ratio. A similar correlation is found to be applicable to heat and mass transfer in packed and fluidized beds and through screens.

A limited amount of data has been obtained on the effect of the Prandtl modulus at high Reynolds numbers. These data seem to indicate, as do those for flow through tubes, that the Prandtl number exponent is a function of Reynolds number.

EFFECT OF VOID VOLUME

The rate of heat transfer for fluids flowing normally to banks of circular tubes is of interest in the design not only of cross-flow exchangers, but also of the more common baffled exchangers. Heat transfer in cross flow has therefore been the subject of a number of investigations. Despite the considerable quantity of data in the literature, there is no generalized correlation relating heat transfer characteristics to lattice geometry. It is the object of the first part of this paper to present such a correlation and to show how this may be applied to packed and fluidized beds.

The first correlation of cross-flow data was made by Colburn (*5*) in 1933. He assigned the letter j to the quantity $(h/CpG_{max})(Pr)^{2/3}$ and found that the data then available for staggered tube banks were represented by

$$j = 0.33\left(\frac{DG_{max}}{\mu_f}\right)^{-0.4}$$

$$\left(\frac{DG_{max}}{\mu_f}\right) > 300 \qquad (1)$$

Since that time the extensive data of Pierson (*23*) and Kays, London, and Lo (*19*) for air in turbulent flow have appeared. These data show that the lattice arrangement has a definite effect on the heat transfer coefficients obtained. Pierson found that the variation due to lattice spacing increased with decreasing Reynolds number. Grimison (*15*) correlated the data of Pierson by

$$\frac{hD}{k_f} = 0.28\,\mathrm{Fa}\left(\frac{DG_{max}}{\mu_f}\right)^{0.61} \qquad (2)$$

where Fa was an empirical factor, depending on lattice arrangement and Reynolds number, which varied from 0.65 to 1.3.

Bergelin et al. (*1*) (*2*) investigated the effect of lattice spacing in the intermediate and laminar regions. The lattice arrangement was found to have a noticeable effect, which was greater at the lower Reynolds numbers. At the lowest Reynolds numbers investigated it was found that there was more than a twofold variation in exchanger performance.

Present Correlation

The variation in the heat transfer characteristics of various lattices can be related to the ratio of the tube pitches and the void fraction, ε, of the lattice. The void fraction is defined as

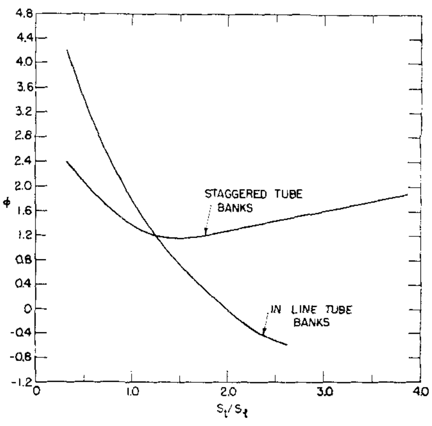

Fig. 1. Variation of arrangement of coefficients with s_t/s_l.

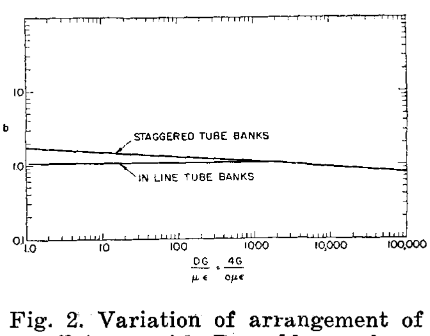

Fig. 2. Variation of arrangement of coefficients with Reynolds number.

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$$\epsilon = \frac{\text{total volume of lattice} - \text{volume of tubes}}{\text{total volume of lattice}} \quad (3)$$

By means of the void-fraction concept it is possible to use a Reynolds number based on the average velocity through the lattice.

$$Re = DG/\mu\epsilon \quad (4)$$

Using this Reynolds number, one may define a j factor as follows:

$$j^1 = \left(\frac{\bar{h}\epsilon}{C_{pf}G}\right)\left(\frac{C_{pf}\mu_f}{k_f}\right)^{\frac{2}{3}} = f(s_t/s_l, \epsilon, Re) \quad (5)$$

or rearranging,

$$j = \left(\frac{\bar{h}}{C_{pf}G}\right)\left(\frac{C_{pf}\mu_f}{k_f}\right)^{\frac{2}{3}} = f(s_t/s_l, \epsilon, Re) \quad (6)$$

Equation (6) will be used since it is in harmony with the treatment now given to packed and fluidized beds. It is found that the effects of void-volume-and-tube-pitch ratio may be accounted for by using an equation of the form

$$j\,\epsilon^{\phi b} = f(Re) \quad (7)$$

where ϕ = arrangement exponent depending on the ratio, s_t/s_l, of transverse to longitudinal tube pitch
b = coefficient depending on the Reynolds number (fixed at 1 for $DG/\mu\epsilon = 4{,}000$).

The values of ϕ and b were obtained by correlating the available data (*1, 2, 19, 23*), which include values of ϵ from 0.415 to 0.91. It was found that the exponents required for in-line arrangements differed from those required for staggered tube banks. Figure 1 shows ϕ as a function of s_t/s_l for both arrangements, and Figure 2 shows b as a function of Reynolds number. For staggered tube banks

$$b = 1.75\,(DG/\mu\epsilon)^{-0.07} \quad (8)$$

For in-line tube banks, b remains constant at a value of 1.1 until a Reynolds number of 1,200 is reached. At higher Reynolds numbers the values of b for both arrangements coincide.

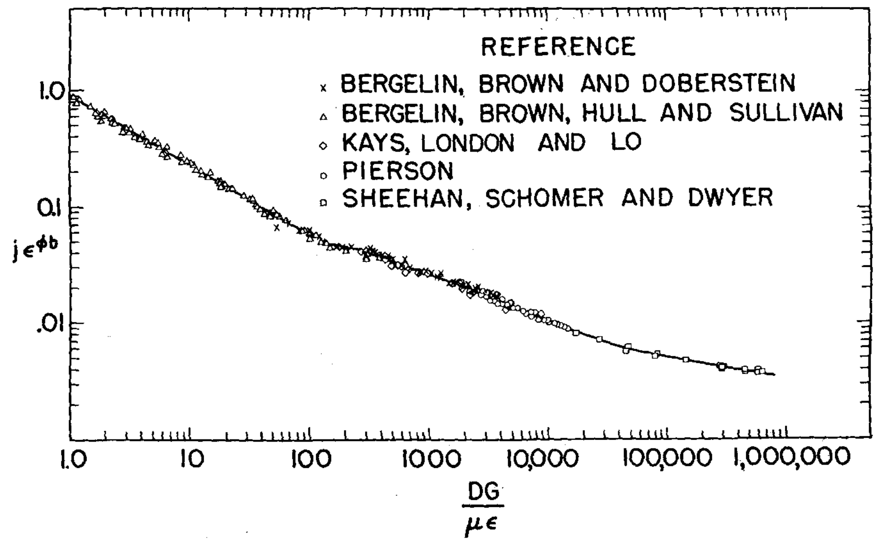

Fig. 3. Correlation of data for heat transfer rates in staggered tube banks.

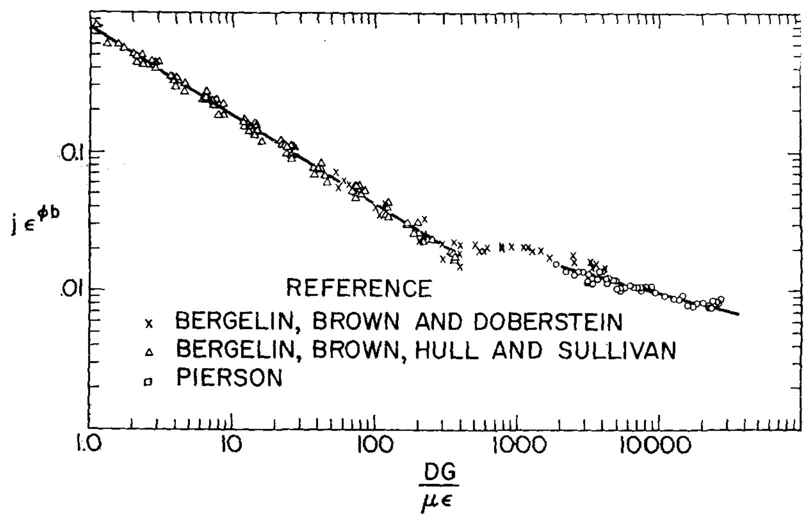

Fig. 4 Correlation of data for heat transfer rates in in-line tube banks.

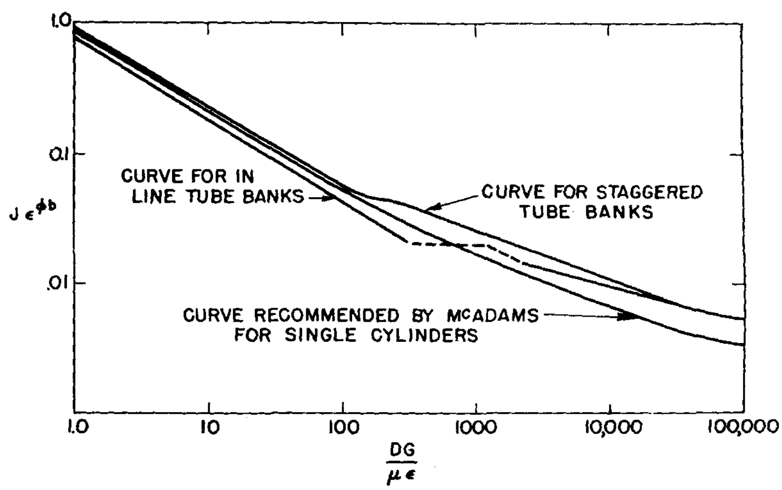

Fig. 5. Comparison of the rate of heat transfer from single cylinders with data obtained in tube banks.

The data of Bergelin and coworkers (*1* and *2*) for staggered tube banks together with points from each set of data for the different staggered arrangements of Pierson (*23*) and Kays, London and Lo (*19*) are plotted in Figure 3. In the laminar and intermediate regions Bergelin et al. found that the effect of physical properties was best accounted for by using $Pr^{\frac{2}{3}}$ together with the Sieder and Tate (*29*) correction. Therefore, in the data of Bergelin and coworkers

$$j = \left(\frac{\bar{h}}{C_p G}\right)\left(\frac{C_{pB}\mu}{k_B}\right)^{\frac{2}{3}}\left(\frac{\mu_W}{\mu}\right)^{0.14} \quad (9)$$

A single smooth curve is obtained from the data of the several investigations. In both the viscous and turbulent regions the curve can be very closely approximated by straight lines on logarithmic coordinates. The equations in these regions are

$$j\,\epsilon^{\phi_b} = 0.895\,(DG/\mu\epsilon)^{-0.59}$$

where

$$(1 < (DG/\mu\epsilon) < 100) \quad (10)$$

$$j\,\epsilon^{\phi_b} = 0.38\,(DG/\mu\epsilon)^{-0.39}$$

where

$$(300 < (DG/\mu\epsilon) < 40{,}000) \quad (11)$$

A very good correlation is found to exist. The lattice spacings investigated by Pierson (*23*) and Kays, London, and Lo (*19*) and Bergelin et al. (*1* and *2*) show no deviation from the curve in excess of 10% although a few individual points show deviations in excess of this.

Although only one lattice arrangement was investigated, the data obtained by Sheehan et al. (*28*) for very high Reynolds numbers are also shown in Figure 3. In the plotting of these data it was assumed that correct values of b could be obtained by extrapolation of Equation (8) to these high Reynolds numbers. By this procedure the following is obtained:

$$j\,\epsilon^{\phi_b} = 0.051\,(DG/\mu\epsilon)^{-0.2}$$

where

$$(DG/\mu\epsilon) > 80{,}000 \quad (12)$$

The data of Bergelin et al. (*1* and *2*) together with representative points from each of the in-line lattices studied by Pierson (*23*) are plotted in Figure 4. The discontinuity between the fully turbulent and laminar regions is more marked in the in-line banks than in staggered tube banks. In the viscous region the data are correlated by

$$j\,\epsilon^{\phi_b} = 0.785\,(DG/\mu\epsilon)^{-0.63}$$

where

$$(1 < (DG/\mu\epsilon) < 300) \quad (13)$$

In the fully turbulent region the following is obtained:

$$j\,\epsilon^{\phi_b} = 0.101\,(DG/\mu\epsilon)^{-0.255}$$

where

$$(2{,}000 < (DG/\mu\epsilon) < 30{,}000) \quad (14)$$

The deviation of the experimental points from the foregoing equations appears to be within the experimental error, as no lattice spacing shows a deviation in excess of 15%.

A comparison of the tube-bank correlations (Figure 5) shows that the staggered-tube data lie above the data for in-line banks until a Reynolds number of 30,000 is reached. From this point on, the two curves appear to be coincident. It will be noted that for the intermediate Reynolds number range there is considerable variation between the curves for in-line and staggered tube banks. The use of a single correlation for both arrangements can lead to errors as high as 90%. Both tube-bank correlations lie above the data for single cylinders in the fully turbulent region. In the laminar region the staggered-tube-bank correlation lies above the single-cylinder data but the in-line-tube-bank correlation lies below. The failure of these correlations to extrapolate to $\epsilon = 1$ indicates that they should not be used for situations where the void fraction, ϵ, is much in excess of 0.9.

Heat and Mass Transfer in Packed and Fluidized Beds

The first problem which presents itself when an attempt is made to extend the correlation developed for tube banks to other systems is the choice of a suitable substitute for tube diameter. A quantity which has been found useful in pressure-drop correlations is the specific surface, ***a***.

$$a = \frac{\text{surface area of particle}}{\text{volume of particle}} \quad (15)$$

The Reynolds number then becomes

$$Re = G/a\,\epsilon\mu \quad (16)$$

For a cylinder $1/a$ equals one fourth the diameter; for a sphere $1/a$ equals one sixth the diameter.

The utility of this concept when applied to heat transfer is illustrated in Figure 6. There the heat and mass transfer data for single spheres in air are compared with the curve recommended by McAdams (*21*) for single cylinders. The agreement between the sphere data and cylinder line is good, particularly for the mass transfer data.

By use of the Reynolds number defined by Equation (16), the data of McCune and Wilhelm (*22*) and Chu et al. (*4*) for fluidized beds of spherical particles may be correlated in the same manner as the tube-bank data. The value of ϕ is found to be 1.1, and b is obtained from the curve used for staggered tube banks. (*See* Figure 2.) When so treated, the mass transfer data are correlated by the curve previously obtained for heat transfer in staggered tube banks. (*See* Figure 7.)

The data of McCune and Wilhelm (*22*) agree very well with the curve but some scatter is observed in the data of Chu et al. (*4*). This scatter appears to be inherent in the data, the poorer precision apparently being caused by the difficulty in measuring the bed height, and therefore ϵ, in gas fluidization. The fluidized-bed data shown do not include data for values of ϵ in excess of 0.86. At higher void fractions the data of McCune and Wilhelm (*22*) deviate from the curve and begin to approach those for single particles.

The data for fixed beds may be handled similarly to those for fluidized beds. However, an additional complication is introduced by the fact that not all the area of the packing or particles is effective in the transfer of mass and heat. Gamson (*13*) has suggested that the j factor be divided by an area factor, representing the fraction of the packing area which is effective. The gas-film data of Hougen, Gamson, and coworkers (*14, 30, 33*) are plotted in Figure 8 by means of the area factors recommended by Gamson (*13*). The value of ϕ for packed beds is also found to be 1.1 and b is again read from the curve for staggered tube banks. (*See* Figure 2.)

Good agreement is obtained between the data for various packing shapes. As seen from Figure 8, above $Re = 10$ the packed data are also correlated by the curve previously obtained for staggered tube banks.

The liquid-film data of various investigators (*7, 10, 12, 13, 22*) for packed beds of spherical particles at low Reynolds numbers are not shown. As pointed out by Dryden, et al. (*7*) in liquid systems at low Reynolds numbers the rate of transfer obtained by packed beds is influenced by free convection, which in turn varies with the Grashof number. It is not to be expected, therefore, that those data can be adequately correlated by use of j factor concept.

Heat Transfer Through Wire-Screen Matrices

The data of Coppage and London (*6*) for heat transfer through beds of wire screens may also be correlated in the manner used for packed and fluidized beds. In this case it is necessary to define ε on the basis of a single screen, as it was found that the spacing between successive screens did not affect the rate of heat transfer. Over the range of Reynolds numbers investigated, the data seemed to be best fitted when ϕb equal to a constant value of 0.8 was used. The data, given in Figure 9, shows good agreement between the various matrices. The points are fitted fairly closely by the curve derived from staggered-tube-bank data (maximum deviation of 30%) but are somewhat better correlated by the dotted line having the following equation:

$$j\,\epsilon^{0.8} = 0.36\,(G/a\,\epsilon\mu)^{-0.5}$$

where

$$(0.5 < (G/a\,\epsilon\mu) < 60) \quad (17)$$

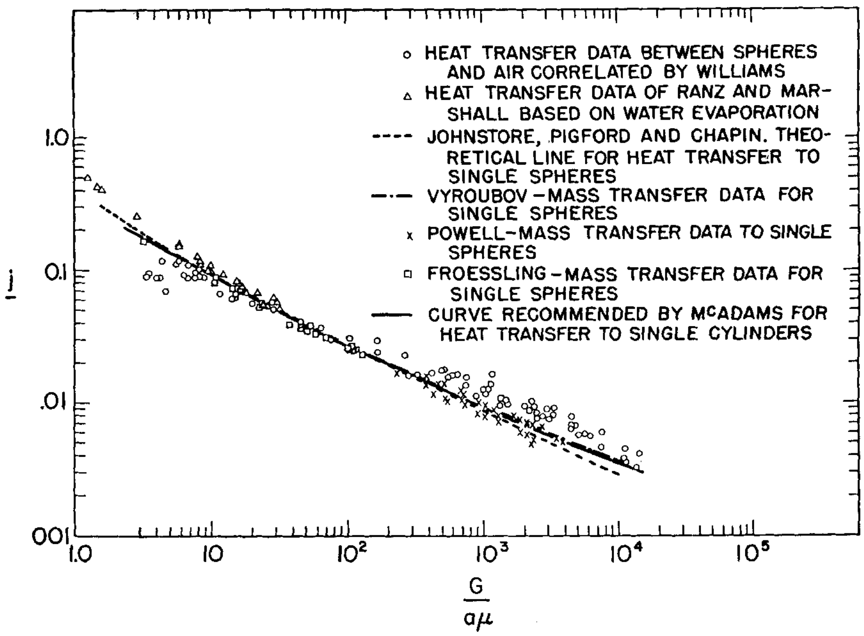

Fig. 6. Comparison of the rate of heat transfer from single cylinders with the data for single spheres.

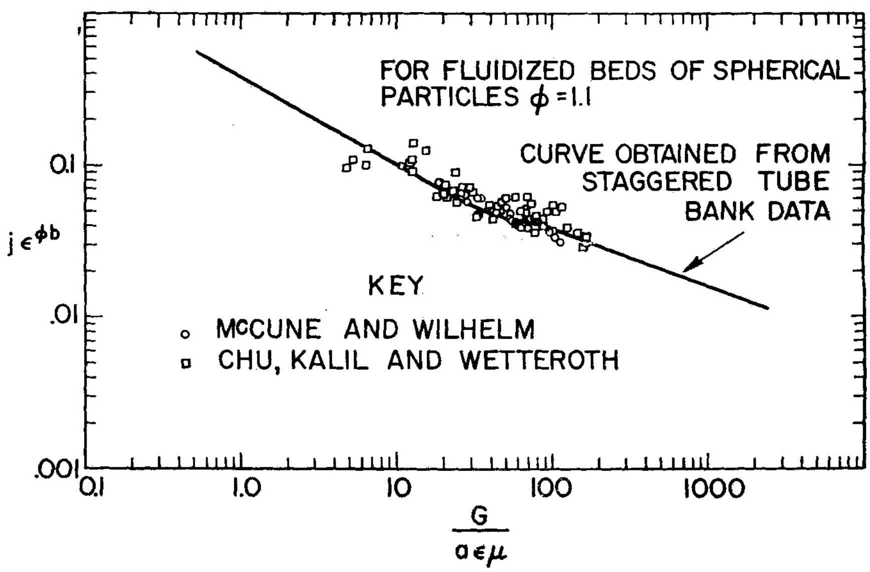

Fig. 7. Comparison of data for beds of fluidized spheres with curve from staggered-tube-bank data.

EFFECT OF PRANDTL NUMBER IN CROSS FLOW

Comparatively little work has been done on the effect of the Prandtl modulus on heat transfer for the case of fluids flowing across tube banks. In general it has been assumed that the effect is the same in cross flow as in the better understood case of flow through tubes. The latter case was first analyzed for the turbulent region by Prandtl (*25*) and Taylor (*31*) and more recently and extensively by Von Karman (*18*), Boelter et al. (*1*), and Reichart (*27*), using velocity-distribution measurements in straight tubes.

Colburn (*5*) showed that over the range of Reynolds numbers then investigated the complex function of Prandtl number appearing in the theoretical derivations could be replaced by the Prandtl number to a constant power. Colburn could thus empirically correlate the data by equations of the form

$$f(Re) = (\bar{h}/C_pG)\,Pr^{2/3} \quad (18)$$

One of the most recent theoretical analyses is due to Lin, Moulton, and Putnam (*20*), who introduced the concept of a small amount of eddy in what is normally considered the laminar layer. With their analysis they were able to obtain considerably better agreement for heat transfer at Prandtl numbers greater than 1 and for mass transfer data at high Schmidt numbers. They presented their results as a series of curves of h/CpG vs. Reynolds number with Prandtl number as parameter. These curves may be very closely approximated by an equation of the form

$$f(Re) = (\bar{h}/C_pG)\,Pr^{n} \quad (19)$$

where n is itself a function of the Reynolds number.

The variation in the Prandtl number exponent is shown in Figure 10.

$$n = 1.22\, Re^{-0.07} \qquad (20)$$

It is seen that the Colburn approximation of $n = 2/3$ is a good average value for use at fairly low Reynolds numbers. However, the deviation from this becomes large at high Reynolds numbers.

One might expect a similar situation to obbtain in cross flow. It would also be expected that the effect would be noticeable at lower Reynolds numbers owing to the greater turbulence in cross flow. A small quantity of data which indicate that this may be true were obtained at Brookhaven National Laboratory as part of the previously reported (*8*, *9*, *28*) extensive investigation of heat ransfer rate for cross flow at high Reynolds numbers.

Both local and average heat transfer coefficients were obtained for two lattice positions at two Reynolds numbers and a Prandtl number of approximately three. The equipment and methods used were those described by Sheehan et al. (*28*) and therefore will not be given in detail here. Briefly the heat transfer coefficients were obtained by measuring the total temperature difference between the bulk water temperature and the inside-wall temperature of a hollow electrically heated nickel tube. The wall temperatures were obtained by means of a revolving pin-point thermocouple probe. The tube bank used consisted of 200 elements 10 rows wide and 20 deep. The elements, 0.810 in. in diameter, were set in an equitriangular spacing with a pitch of 1 9/32 in.

It is found that the 2/3 power of the Prandtl number does not bring the data taken at $Pr \cong 3$ into agreement with the $Pr \cong 1$ data. The discrepancy is found to exist whether the comparison is based on the use of the average film temperature for the determination of fluid properties or on the use of bulk water temperatures together with the Sieder and Tate (*29*) correction. If it is assumed that the effect of the Prandtl number in cross flow is similar to the effect in flow through tubes, this discrepancy can be explained. It is known that for the turbulent region, $n = 2/3$ is a good average value between approximately $(DG/\mu\varepsilon) = 100$ and $(DG/\mu\varepsilon) = 2{,}000$. If

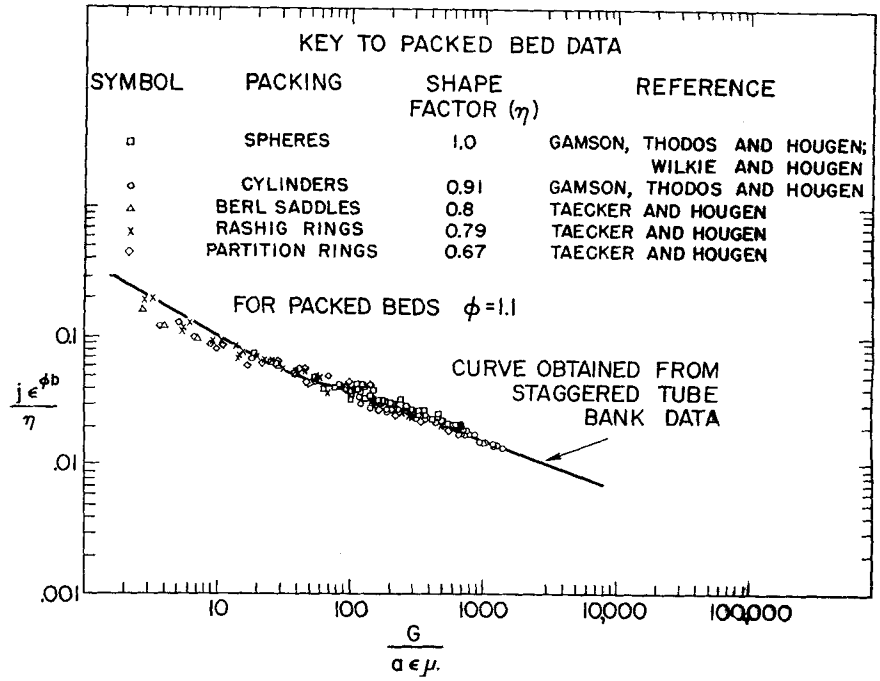

Fig. 8. Correlation of packed-bed heat transfer data.

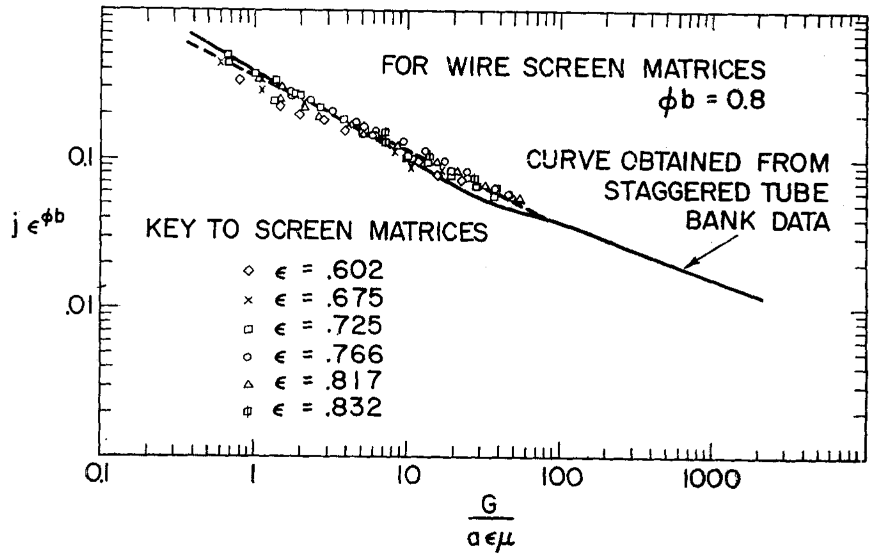

Fig. 9. Correlation of wire-screen heat transfer data.

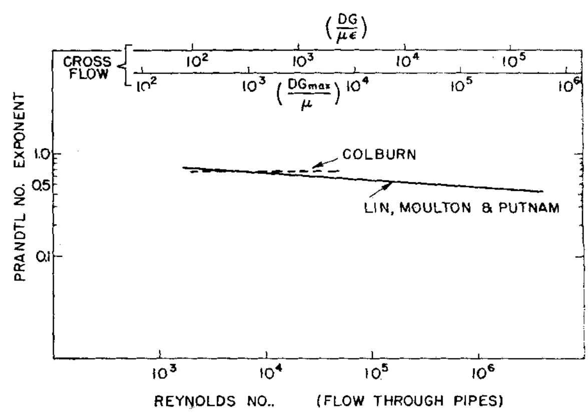

Fig. 10. Variable Prandtl number exponent obtained from work of Lin et al. (*20*).

a line parallel to Equation (20) is drawn through this region, one obtains for staggered tube banks

$$n = 0.985\,(DG/\mu\epsilon)^{-0.07} \qquad (21)$$

In Figures 11 and 12 the experimental values for the average film coefficients are compared with the lines obtained by extrapolation of the $Pr = 1$ data for the same lattice positions. It is seen that the line obtained by use of the 2/3 power of Prandtl number falls considerably below the experimental points; however, the line obtained with the variable exponent of Equation (21) appears to fit the data within the experimental error. For the range of Reynolds numbers explored the Prandtl number exponent is about 0.48.

In Figures 13 and 14 a comparison similar to the foregoing is made for the local heat transfer coefficients. The experimental points are compared with the curves obtained by extrapolation of the local-coefficients data of Dwyer, Sheehan, and Weisman (*9*). Again good agreement is obtained with the prediction based on the variable Prandtl number exponent of Equation (21).

It is realized that the results given here must be regarded as tentative in view of the small amount of data available. It is regrettable that budgetary restrictions prevented additional work at other Reynolds and Prandtl numbers, and it is hoped that other workers will investigate this problem.

NOTATION

a = surface area of particle per unit volume of particle, sq.ft./cu.ft.
b = Reynolds number exponent, dimensionless
C_p = specific heat, B.t.u./(lb.) (°F.)
D = diameter of cylinder, ft.
Fa = arrangement of factor of Grimison, dimensionless
G = superficial mass velocity based on total cross-sectional area, lb./(hr.) (sq.ft.)
G_{max} = maximum mass velocity based on minimum flow area, lb./(hr.) (sq.ft.)
h = local heat transfer coefficient for a single point on the circumference of a tube, B.t.u./(hr.) (sq.ft.) (°F.)
h = average heat transfer coefficient, B.t.u./(hr.) (sq.ft.) (°F.)
j = transfer coefficient, dimensionless for heat transfer $j = (h/C_{pf}G)\,(Pr)^{2/3}$ for mass transfer $j = (K'\rho/G)\,(Sc)^{2/3}$
k = thermal conductivity of fluid, B.t.u./(hr.) (sq.ft.) (°F.)/ft.
K' = mass transfer coefficient in concentration units, moles/(hr.) (sq.ft.) (moles/cu.ft.)
n = Prandtl number exponent, dimentionless
Pr = Prandtl number, dimensionless
Re = Reynolds number, dimensionless
s_l = longitudinal tube pitch (distance between tube centers in direction parallel to flow)
s_t = transverse tube pitch (distance between tube centers in direction normal to flow)
S_c = Schmidt number, dimensionless
ϵ = void fraction, dimensionless
ρ = density, lb./cu.ft.
ϕ = arrangement factor, dimensionless
η = fraction of packing area effective in heat and mass transfer, dimensionless
μ = viscosity, lb./(ft.) (hr.) (evaluated at bulk temperature when no subscript is used)

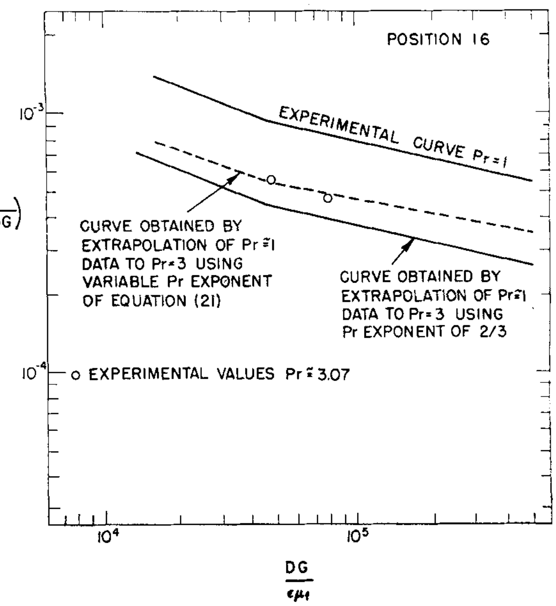

Fig. 11. Comparison of experimental values of the average coefficients (central position) at $Pr = 3$ with curves extrapolated from $Pr = 1$ data.

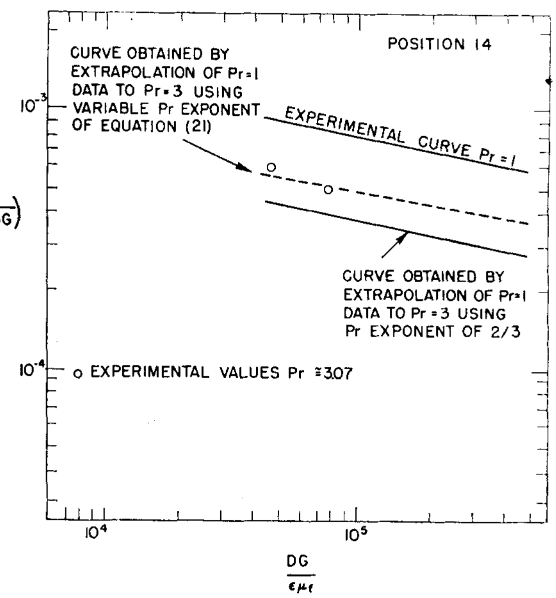

Fig. 12. Comparison of experimental values of the average coefficients (wall position) at $Pr = 3$ with curves extrapolated from $Pr = 1$ data.

Subscripts

B = evaluated at bulk temperature
f = evaluated at film temperature
W = evaluated at wall temperature

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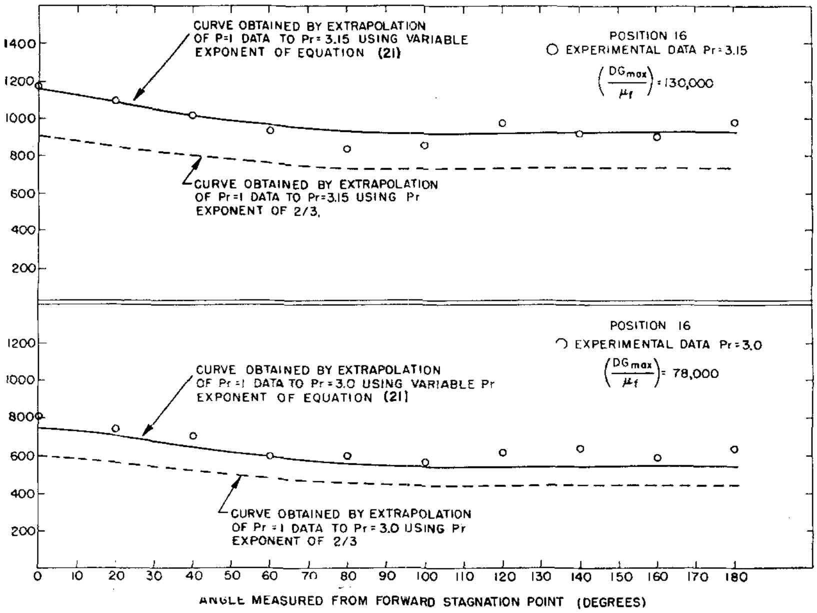

Fig. 13. Comparison of experimental values of individual film coefficients (central position) at $Pr = 3$ with curves extrapolated from $Pr = 1$ data.

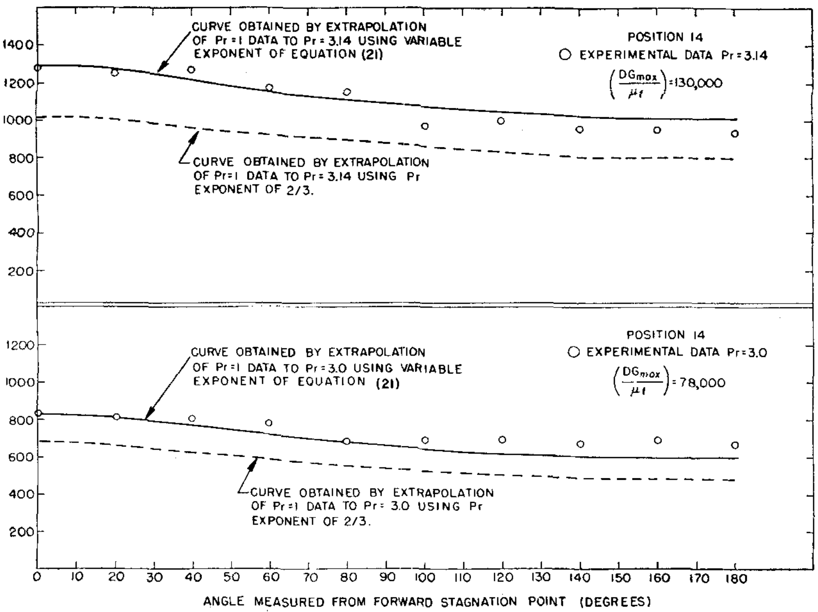

Fig. 14 Comparison of experimental values of individual film coefficients (wall position) at $Pr = 3$ with curves extrapolated from $Pr = 1$ data.

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